1. a) $\Delta c_A = 2{,}5 - 3 = -0{,}5 \in [-1; 3]$. Then, the OS does not change and the daily cost decreases 0,5 m.u. : $\Delta Z = \Delta c_A \times x_A = -0{,}5 \times 1 = -0{,}5$.

1. b) $\Delta b_3 = 5 - 3 = 2 \in [-3, 2]$ and the daily cost will decrease 2 m.: u. $\Delta Z = \Delta b_3\, y_3 = 2 \times (-1) = -2$.

1. c) The daily cost is $Z = 3 \times 1 + 4 \times 1 + 3 = 10\, m.u.$ In 10 days the cost will be $100\, m.u. < 105\, m.u.$ So the budget is enough.

1. d) Let: y_j = number of trips to transport forage j ($j = \mathbf{A}, \mathbf{B}$). The changes in the model should be:

New objective function to minimize: $Z = 3\, x_A + 4\, x_B + x_P + 150 y_A + 150 y_B$

Additional constraints: $\begin{cases} x_j \le \frac{1}{2}\, y_j & j = A, B \\ y_j \ge 0 \text{ e integer } j = A, B \end{cases}$

2. Minimal spanning tree as the scope is to connect **all** strategic sites. Prim Algorithm, $n - 1 = 5$ iterations:

Iteration	Node ∈ tree	Closest Node ∉ tree	Min distance	edge to include in the tree
1	A	D	**30**	(A, D)
2	A	E	50	
	D	F	**30**	(D, F)
3	A	E	50	
	D	B	50	
	F	C	**20**	(F,C)
4	A	E	50	
	D	B	50	
	F	E	60	
	C	B	**40**	(C,B)
5	A	E	**50**	(A,E)
	D	-	-	
	F	E	60	
	C	-	-	
	B	-	-	

Total Cost= $200 \times (30 + 30 + 20 + 40 + 50) = 34000$€. So the municipality should spend in addition 14000€ $(34000 - 20000)$.

3. TP unbalanced as: Total Supply $= 3 \times 3000 = 9000 > 2 \times 4000 = 8000 =$ Total Demand. Let $x_{ij} = m^3$ of water to transport from reservoir Ri ($i = 3,4,5$) to reservoir Rj ($j = 1,2$),

$$Min\ Z = 5\, x_{31} + 3\, x_{32} + 2\, x_{41} + 5\, x_{42} + 2\, x_{51} + 3\, x_{52}$$

$$\text{s.t.:} \begin{cases} x_{i1} + x_{i2} \le 3000 & i = 3,4,5 \\ x_{3j} + x_{4j} + x_{5j} = 4000 & j = 1,2 \\ x_{ij} \ge 0 & i = 3,4,5;\ j = 1,2 \end{cases}$$